Chương 2: Khái niệm cơ bản về trường điện từ

Bài 1:

2.5. Let a point charge $Q_1 25$ nC be located at $P_1(4, -2, 7)$ and a charge $Q_2 = 60$ nC be at $P_2(-3, 4, -2)$.

a) If $\epsilon = \epsilon_0$, find $\mathbf{E}$ at $P_3(1, 2, 3)$: This field will be

$$\mathbf{E} = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25\mathbf{R}_{13}}{|\mathbf{R}_{13}|^3} + \frac{60\mathbf{R}_{23}}{|\mathbf{R}_{23}|^3}\right]$$

where $\mathbf{R}_{13} = -3\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z$ and $\mathbf{R}_{23} = 4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z$. Also, $|\mathbf{R}_{13}| = \sqrt{41}$ and $|\mathbf{R}_{23}| = \sqrt{45}$. So

$$\mathbf{E} = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25 \times (-3\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z)}{(41)^{1.5}} + \frac{60 \times (4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z)}{(45)^{1.5}}\right]$$
$$= \underline{4.58\mathbf{a}_x - 0.15\mathbf{a}_y + 5.51\mathbf{a}_z}$$

b) At what point on the y axis is $E_x = 0$? P_3 is now at $(0, y, 0)$, so $\mathbf{R}_{13} = -4\mathbf{a}_x + (y+2)\mathbf{a}_y - 7\mathbf{a}_z$ and $\mathbf{R}_{23} = 3\mathbf{a}_x + (y-4)\mathbf{a}_y + 2\mathbf{a}_z$. Also, $|\mathbf{R}_{13}| = \sqrt{65 + (y+2)^2}$ and $|\mathbf{R}_{23}| = \sqrt{13 + (y-4)^2}$. Now the x component of $\mathbf{E}$ at the new P_3 will be:

$$E_x = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25 \times (-4)}{[65 + (y+2)^2]^{1.5}} + \frac{60 \times 3}{[13 + (y-4)^2]^{1.5}}\right]$$

To obtain $E_x = 0$, we require the expression in the large brackets to be zero. This expression simplifies to the following quadratic:

$$0.48y^2 + 13.92y + 73.10 = 0$$

which yields the two values: $y = \underline{-6.89, -22.11}$

Bài 2:

Point charges of 120 nC are located at $A(0, 0, 1)$ and $B(0, 0, -1)$ in free space.

a) Find **E** at $P(0.5, 0, 0)$: This will be

$$\mathbf{E}_P = \frac{120 \times 10^{-9}}{4\pi\epsilon_0}\left[\frac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3} + \frac{\mathbf{R}_{BP}}{|\mathbf{R}_{BP}|^3}\right]$$

where $\mathbf{R}_{AP} = 0.5\mathbf{a}_x - \mathbf{a}_z$ and $\mathbf{R}_{BP} = 0.5\mathbf{a}_x + \mathbf{a}_z$. Also, $|\mathbf{R}_{AP}| = |\mathbf{R}_{BP}| = \sqrt{1.25}$. Thus:

$$\mathbf{E}_P = \frac{120 \times 10^{-9}\mathbf{a}_x}{4\pi(1.25)^{1.5}\epsilon_0} = \underline{772 \text{ V/m}}$$

b) What single charge at the origin would provide the identical field strength? We require

$$\frac{Q_0}{4\pi\epsilon_0(0.5)^2} = 772$$

from which we find $Q_0 = \underline{21.5 \text{ nC}}$.

Bài 3:

2.7. A 2 μC point charge is located at $A(4, 3, 5)$ in free space. Find E_ρ, E_ϕ, and E_z at $P(8, 12, 2)$. Have

$$\mathbf{E}_P = \frac{2 \times 10^{-6}}{4\pi\epsilon_0}\frac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3} = \frac{2 \times 10^{-6}}{4\pi\epsilon_0}\left[\frac{4\mathbf{a}_x + 9\mathbf{a}_y - 3\mathbf{a}_z}{(106)^{1.5}}\right] = 65.9\mathbf{a}_x + 148.3\mathbf{a}_y - 49.4\mathbf{a}_z$$

Then, at point P, $\rho = \sqrt{8^2 + 12^2} = 14.4$, $\phi = \tan^{-1}(12/8) = 56.3°$, and $z = z$. Now,

$$E_\rho = \mathbf{E}_p \cdot \mathbf{a}_\rho = 65.9(\mathbf{a}_x \cdot \mathbf{a}_\rho) + 148.3(\mathbf{a}_y \cdot \mathbf{a}_\rho) = 65.9\cos(56.3°) + 148.3\sin(56.3°) = \underline{159.7}$$

and

$$E_\phi = \mathbf{E}_p \cdot \mathbf{a}_\phi = 65.9(\mathbf{a}_x \cdot \mathbf{a}_\phi) + 148.3(\mathbf{a}_y \cdot \mathbf{a}_\phi) = -65.9\sin(56.3°) + 148.3\cos(56.3°) = \underline{27.4}$$

Finally, $E_z = \underline{-49.4 \text{ V/m}}$

Bài 4:

2.11. A charge Q_0 located at the origin in free space produces a field for which $E_z = 1$ kV/m at point $P(-2, 1, -1)$.

a) Find Q_0: The field at P will be

$$\mathbf{E}_P = \frac{Q_0}{4\pi\epsilon_0}\left[\frac{-2\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z}{6^{1.5}}\right]$$

Since the z component is of value 1 kV/m, we find $Q_0 = -4\pi\epsilon_0 6^{1.5} \times 10^3 = \underline{-1.63\ \mu\text{C}}$.

b) Find $\mathbf{E}$ at $M(1, 6, 5)$ in cartesian coordinates: This field will be:

$$\mathbf{E}_M = \frac{-1.63 \times 10^{-6}}{4\pi\epsilon_0}\left[\frac{\mathbf{a}_x + 6\mathbf{a}_y + 5\mathbf{a}_z}{[1 + 36 + 25]^{1.5}}\right]$$

or $\mathbf{E}_M = \underline{-30.11\mathbf{a}_x - 180.63\mathbf{a}_y - 150.53\mathbf{a}_z}$.

c) Find $\mathbf{E}$ at $M(1, 6, 5)$ in cylindrical coordinates: At M, $\rho = \sqrt{1 + 36} = 6.08$, $\phi = \tan^{-1}(6/1) = 80.54°$, and $z = 5$. Now

$$E_\rho = \mathbf{E}_M \cdot \mathbf{a}_\rho = -30.11 \cos\phi - 180.63 \sin\phi = -183.12$$

$$E_\phi = \mathbf{E}_M \cdot \mathbf{a}_\phi = -30.11(-\sin\phi) - 180.63\cos\phi = 0 \quad \text{(as expected)}$$

so that $\mathbf{E}_M = -183.12\mathbf{a}_\rho - 150.53\mathbf{a}_z$.

d) Find $\mathbf{E}$ at $M(1, 6, 5)$ in spherical coordinates: At M, $r = \sqrt{1 + 36 + 25} = 7.87$, $\phi = 80.54°$ (as before), and $\theta = \cos^{-1}(5/7.87) = 50.58°$. Now, since the charge is at the origin, we expect to obtain only a radial component of $\mathbf{E}_M$. This will be:

$$E_r = \mathbf{E}_M \cdot \mathbf{a}_r = -30.11 \sin\theta\cos\phi - 180.63 \sin\theta\sin\phi - 150.53\cos\theta = \underline{-237.1}$$

Bài 5:

2.13. A uniform volume charge density of 0.2 μC/m^3 is present throughout the spherical shell extending from $r = 3$ cm to $r = 5$ cm. If $\rho_v = 0$ elsewhere:

a) find the total charge present throughout the shell: This will be

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_{.03}^{.05} 0.2\, r^2 \sin\theta\, dr\, d\theta\, d\phi = \left[4\pi(0.2)\frac{r^3}{3}\right]_{.03}^{.05} = 8.21 \times 10^{-5}\ \mu\text{C} = \underline{82.1\ \text{pC}}$$

b) find r_1 if half the total charge is located in the region $3\,\text{cm} < r < r_1$: If the integral over r in part a is taken to r_1, we would obtain

$$\left[4\pi(0.2)\frac{r^3}{3}\right]_{.03}^{r_1} = 4.105 \times 10^{-5}$$

Thus

$$r_1 = \left[\frac{3 \times 4.105 \times 10^{-5}}{0.2 \times 4\pi} + (.03)^3\right]^{1/3} = \underline{4.24\ \text{cm}}$$

Bài 6:

2.17. A uniform line charge of 16 nC/m is located along the line defined by $y = -2$, $z = 5$. If $\epsilon = \epsilon_0$:

a) Find **E** at $P(1, 2, 3)$: This will be

$$\mathbf{E}_P = \frac{\rho_l}{2\pi\epsilon_0} \frac{\mathbf{R}_P}{|\mathbf{R}_P|^2}$$

where $\mathbf{R}_P = (1, 2, 3) - (1, -2, 5) = (0, 4, -2)$, and $|\mathbf{R}_P|^2 = 20$. So

$$\mathbf{E}_P = \frac{16 \times 10^{-9}}{2\pi\epsilon_0} \left[\frac{4\mathbf{a}_y - 2\mathbf{a}_z}{20} \right] = \underline{57.5\mathbf{a}_y - 28.8\mathbf{a}_z \text{ V/m}}$$

b) Find **E** at that point in the $z = 0$ plane where the direction of **E** is given by $(1/3)\mathbf{a}_y - (2/3)\mathbf{a}_z$: With $z = 0$, the general field will be

$$\mathbf{E}_{z=0} = \frac{\rho_l}{2\pi\epsilon_0} \left[\frac{(y+2)\mathbf{a}_y - 5\mathbf{a}_z}{(y+2)^2 + 25} \right]$$

We require $|E_z| = -|2E_y|$, so $2(y + 2) = 5$. Thus $y = 1/2$, and the field becomes:

$$\mathbf{E}_{z=0} = \frac{\rho_l}{2\pi\epsilon_0} \left[\frac{2.5\mathbf{a}_y - 5\mathbf{a}_z}{(2.5)^2 + 25} \right] = \underline{23\mathbf{a}_y - 46\mathbf{a}_z}$$

Bài 7:

2.19. A uniform line charge of 2 μC/m is located on the z axis. Find **E** in cartesian coordinates at $P(1, 2, 3)$ if the charge extends from

a) $-\infty < z < \infty$: With the infinite line, we know that the field will have only a radial component in cylindrical coordinates (or x and y components in cartesian). The field from an infinite line on the z axis is generally $\mathbf{E} = [\rho_l/(2\pi\epsilon_0\rho)]\mathbf{a}_\rho$. Therefore, at point P:

$$\mathbf{E}_P = \frac{\rho_l}{2\pi\epsilon_0} \frac{\mathbf{R}_{zP}}{|\mathbf{R}_{zP}|^2} = \frac{(2 \times 10^{-6})}{2\pi\epsilon_0} \frac{\mathbf{a}_x + 2\mathbf{a}_y}{5} = \underline{7.2\mathbf{a}_x + 14.4\mathbf{a}_y \text{ kV/m}}$$

where $\mathbf{R}_{zP}$ is the vector that extends from the line charge to point P, and is perpendicular to the z axis; i.e., $\mathbf{R}_{zP} = (1, 2, 3) - (0, 0, 3) = (1, 2, 0)$.

b) $-4 \le z \le 4$: Here we use the general relation

$$\mathbf{E}_P = \int \frac{\rho_l dz}{4\pi\epsilon_0} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

where $\mathbf{r} = \mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{r}' = z\mathbf{a}_z$. So the integral becomes

$$\mathbf{E}_P = \frac{(2 \times 10^{-6})}{4\pi\epsilon_0} \int_{-4}^{4} \frac{\mathbf{a}_x + 2\mathbf{a}_y + (3 - z)\mathbf{a}_z}{[5 + (3 - z)^2]^{1.5}} \, dz$$

Using integral tables, we obtain:

$$\mathbf{E}_P = 3597 \left[\frac{(\mathbf{a}_x + 2\mathbf{a}_y)(z - 3) + 5\mathbf{a}_z}{(z^2 - 6z + 14)} \right]_{-4}^{4} \text{ V/m} = \underline{4.9\mathbf{a}_x + 9.8\mathbf{a}_y + 4.9\mathbf{a}_z \text{ kV/m}}$$

Bài 8:

2.23. Given the surface charge density, $\rho_s = 2\,\mu\text{C/m}^2$, in the region $\rho < 0.2$ m, $z = 0$, and is zero elsewhere, find **E** at:

a) $P_A(\rho = 0, z = 0.5)$: First, we recognize from symmetry that only a z component of **E** will be present. Considering a general point z on the z axis, we have $\mathbf{r} = z\mathbf{a}_z$. Then, with $\mathbf{r}' = \rho\mathbf{a}_\rho$, we obtain $\mathbf{r} - \mathbf{r}' = z\mathbf{a}_z - \rho\mathbf{a}_\rho$. The superposition integral for the z component of **E** will be:

$$E_{z,P_A} = \frac{\rho_s}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{0.2}\frac{z\,\rho\,d\rho\,d\phi}{(\rho^2+z^2)^{1.5}} = -\frac{2\pi\rho_s}{4\pi\epsilon_0}z\left[\frac{1}{\sqrt{z^2+\rho^2}}\right]_0^{0.2}$$
$$= \frac{\rho_s}{2\epsilon_0}z\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2+0.04}}\right]$$

With $z = 0.5$ m, the above evaluates as $E_{z,P_A} = \underline{8.1 \text{ kV/m}}$.

b) With z at -0.5 m, we evaluate the expression for E_z to obtain $E_{z,P_B} = \underline{-8.1 \text{ kV/m}}$.

Bài 9:

2.25. Find **E** at the origin if the following charge distributions are present in free space: point charge, 12 nC at $P(2,0,6)$; uniform line charge density, 3nC/m at $x = -2$, $y = 3$; uniform surface charge density, $0.2\,\text{nC/m}^2$ at $x = 2$. The sum of the fields at the origin from each charge in order is:

$$\mathbf{E} = \left[\frac{(12\times10^{-9})}{4\pi\epsilon_0}\frac{(-2\mathbf{a}_x - 6\mathbf{a}_z)}{(4+36)^{1.5}}\right] + \left[\frac{(3\times10^{-9})}{2\pi\epsilon_0}\frac{(2\mathbf{a}_x - 3\mathbf{a}_y)}{(4+9)}\right] - \left[\frac{(0.2\times10^{-9})\mathbf{a}_x}{2\epsilon_0}\right]$$
$$= \underline{-3.9\mathbf{a}_x - 12.4\mathbf{a}_y - 2.5\mathbf{a}_z \text{ V/m}}$$

Chương 3: Dịch chuyển điện – Luật Gauss – Dive

Bài 1:

3.2. A point charge of 20 nC is located at (4,-1,3), and a uniform line charge of -25 nC/m is lies along the intersection of the planes $x = -4$ and $z = 6$.

a) Calculate **D** at (3,-1,0):

The total flux density at the desired point is

$$\mathbf{D}(3,-1,0) = \underbrace{\frac{20 \times 10^{-9}}{4\pi(1+9)} \left[\frac{-\mathbf{a}_x - 3\,\mathbf{a}_z}{\sqrt{1+9}} \right]}_{\text{point charge}} - \underbrace{\frac{25 \times 10^{-9}}{2\pi\sqrt{49+36}} \left[\frac{7\,\mathbf{a}_x - 6\,\mathbf{a}_z}{\sqrt{49+36}} \right]}_{\text{line charge}}$$

$$= -0.38\,\mathbf{a}_x + 0.13\,\mathbf{a}_z \text{ nC/m}^2$$

b) How much electric flux leaves the surface of a sphere of radius 5, centered at the origin? This will be equivalent to how much charge lies within the sphere. First the point charge is at distance from the origin given by $R_p = \sqrt{16+1+9} = 5.1$, and so it is outside. Second, the nearest point on the line charge to the origin is at distance $R_\ell = \sqrt{16+36} = 7.2$, and so the entire line charge is also outside the sphere. Answer: zero.

c) Repeat part *b* if the radius of the sphere is 10.

First, from part *b*, the point charge will now lie inside. Second, the length of line charge that lies inside the sphere will be given by $2y_0$, where y_0 satisfies the equation, $\sqrt{16 + y_0^2 + 36} = 10$. Solve to find $y_0 = 6.93$, or $2y_0 = 13.86$. The total charge within the sphere (and the net outward flux) is now

$$\Phi = Q_{encl} = [20 - (25 \times 13.86)] = -326 \text{ nC}$$

Bài 2:

Bài 3:

3.3. The cylindrical surface $\rho = 8$ cm contains the surface charge density, $\rho_s = 5e^{-20|z|}$ nC/m^2.

a) What is the total amount of charge present? We integrate over the surface to find:

$$Q = 2\int_0^\infty \int_0^{2\pi} 5e^{-20z}(.08)d\phi\, dz \text{ nC} = 20\pi(.08)\left(\frac{-1}{20}\right) e^{-20z}\Big|_0^\infty = \underline{0.25 \text{ nC}}$$

b) How much flux leaves the surface $\rho = 8$ cm, 1 cm $< z <$ 5cm, $30° < \phi < 90°$? We just integrate the charge density on that surface to find the flux that leaves it.

$$\Phi = Q' = \int_{.01}^{.05} \int_{30°}^{90°} 5e^{-20z}(.08)\, d\phi\, dz \text{ nC} = \left(\frac{90-30}{360}\right) 2\pi(5)(.08)\left(\frac{-1}{20}\right) e^{-20z}\Big|_{.01}^{.05}$$

$$= 9.45 \times 10^{-3} \text{ nC} = \underline{9.45 \text{ pC}}$$

Bài 4:

Bài 5:

3.5. Let $\mathbf{D} = 4xy\mathbf{a}_x + 2(x^2 + z^2)\mathbf{a}_y + 4yz\mathbf{a}_z$ C/m^2 and evaluate surface integrals to find the total charge enclosed in the rectangular parallelepiped $0 < x < 2$, $0 < y < 3$, $0 < z < 5$ m: Of the 6 surfaces to consider, only 2 will contribute to the net outward flux. Why? First consider the planes at $y = 0$ and 3. The y component of $\mathbf{D}$ will penetrate those surfaces, but will be inward at $y = 0$ and outward at $y = 3$, while having the same magnitude in both cases. These fluxes

will thus cancel. At the $x = 0$ plane, $D_x = 0$ and at the $z = 0$ plane, $D_z = 0$, so there will be no flux contributions from these surfaces. This leaves the 2 remaining surfaces at $x = 2$ and $z = 5$. The net outward flux becomes:

$$\Phi = \int_0^5 \int_0^3 \mathbf{D}\Big|_{x=2} \cdot \mathbf{a}_x \, dy \, dz + \int_0^3 \int_0^2 \mathbf{D}\Big|_{z=5} \cdot \mathbf{a}_z \, dx \, dy$$
$$= 5 \int_0^3 4(2)y \, dy \; + \; 2 \int_0^3 4(5)y \, dy = \underline{360 \text{ C}}$$

Bài 6:

3.7. Volume charge density is located in free space as $\rho_v = 2e^{-1000r}$ nC/m^3 for $0 < r < 1$ mm, and $\rho_v = 0$ elsewhere.

a) Find the total charge enclosed by the spherical surface $r = 1$ mm: To find the charge we integrate:

$$Q = \int_0^{2\pi} \int_0^{\pi} \int_0^{.001} 2e^{-1000r} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Integration over the angles gives a factor of 4π. The radial integration we evaluate using tables; we obtain

$$Q = 8\pi \left[\frac{-r^2 e^{-1000r}}{1000} \Big|_0^{.001} + \frac{2}{1000} \frac{e^{-1000r}}{(1000)^2} (-1000r - 1) \Big|_0^{.001} \right] = \underline{4.0 \times 10^{-9} \, \text{nC}}$$

b) By using Gauss's law, calculate the value of D_r on the surface $r = 1$ mm: The gaussian surface is a spherical shell of radius 1 mm. The enclosed charge is the result of part a. We thus write $4\pi r^2 D_r = Q$, or

$$D_r = \frac{Q}{4\pi r^2} = \frac{4.0 \times 10^{-9}}{4\pi(.001)^2} = \underline{3.2 \times 10^{-4} \text{ nC/m}^2}$$

Bài 7:

3.9. A uniform volume charge density of $80\,\mu\text{C/m}^3$ is present throughout the region $8\,\text{mm} < r < 10\,\text{mm}$. Let $\rho_v = 0$ for $0 < r < 8\,\text{mm}$.

a) Find the total charge inside the spherical surface $r = 10$ mm: This will be

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_{.008}^{.010} (80\times 10^{-6})r^2 \sin\theta\, dr\, d\theta\, d\phi = 4\pi \times (80\times 10^{-6})\frac{r^3}{3}\Big|_{.008}^{.010}$$
$$= 1.64\times 10^{-10}\,\text{C} = \underline{164\,\text{pC}}$$

b) Find D_r at $r = 10$ mm: Using a spherical gaussian surface at $r = 10$, Gauss' law is written as $4\pi r^2 D_r = Q = 164\times 10^{-12}$, or

$$D_r(10\,\text{mm}) = \frac{164\times 10^{-12}}{4\pi(.01)^2} = 1.30\times 10^{-7}\,\text{C/m}^2 = \underline{130\,\text{nC/m}^2}$$

c) If there is no charge for $r > 10$ mm, find D_r at $r = 20$ mm: This will be the same computation as in part *b*, except the gaussian surface now lies at 20 mm. Thus

$$D_r(20\,\text{mm}) = \frac{164\times 10^{-12}}{4\pi(.02)^2} = 3.25\times 10^{-8}\,\text{C/m}^2 = \underline{32.5\,\text{nC/m}^2}$$

Bài 8:

3.11. In cylindrical coordinates, let $\rho_v = 0$ for $\rho < 1$ mm, $\rho_v = 2\sin(2000\pi\rho)$ nC/m^3 for $1\,\text{mm} < \rho < 1.5\,\text{mm}$, and $\rho_v = 0$ for $\rho > 1.5\,\text{mm}$. Find **D** everywhere: Since the charge varies only with radius, and is in the form of a cylinder, symmetry tells us that the flux density will be radially-directed and will be constant over a cylindrical surface of a fixed radius. Gauss' law applied to such a surface of unit length in z gives:

a) for $\rho < 1$ mm, $\underline{D_\rho = 0}$, since no charge is enclosed by a cylindrical surface whose radius lies within this range.

b) for $1\,\text{mm} < \rho < 1.5\,\text{mm}$, we have

$$2\pi\rho D_\rho = 2\pi\int_{.001}^{\rho} 2\times 10^{-9}\sin(2000\pi\rho')\rho'\,d\rho'$$
$$= 4\pi\times 10^{-9}\left[\frac{1}{(2000\pi)^2}\sin(2000\pi\rho) - \frac{\rho}{2000\pi}\cos(2000\pi\rho)\right]_{.001}^{\rho}$$

or finally,

$$D_\rho = \frac{10^{-15}}{2\pi^2\rho}\left[\sin(2000\pi\rho) + 2\pi\left[1 - 10^3\rho\cos(2000\pi\rho)\right]\right]\,\text{C/m}^2 \quad (1\,\text{mm} < \rho < 1.5\,\text{mm})$$

3.11. (continued)

c) for $\rho > 1.5$ mm, the gaussian cylinder now lies at radius ρ *outside* the charge distribution, so the integral that evaluates the enclosed charge now includes the entire charge distribution. To accomplish this, we change the upper limit of the integral of part b from ρ to 1.5 mm, finally obtaining:

$$D_\rho = \frac{2.5 \times 10^{-15}}{\pi \rho} \ \mathrm{C/m^2} \quad (\rho > 1.5\,\mathrm{mm})$$

Bài 9:

3.13. Spherical surfaces at $r = 2$, 4, and 6 m carry uniform surface charge densities of 20 nC/m^2, $-4\,\mathrm{nC/m^2}$, and ρ_{s0}, respectively.

a) Find **D** at $r = 1$, 3 and 5 m: Noting that the charges are spherically-symmetric, we ascertain that **D** will be radially-directed and will vary only with radius. Thus, we apply Gauss' law to spherical shells in the following regions: $r < 2$: Here, no charge is enclosed, and so $\underline{D_r = 0}$.

$$2 < r < 4: \quad 4\pi r^2 D_r = 4\pi(2)^2(20 \times 10^{-9}) \quad \Rightarrow \quad D_r = \frac{80 \times 10^{-9}}{r^2} \ \mathrm{C/m^2}$$

So $D_r(r = 3) = \underline{8.9 \times 10^{-9}\,\mathrm{C/m^2}}$.

$$4 < r < 6: \quad 4\pi r^2 D_r = 4\pi(2)^2(20 \times 10^{-9}) + 4\pi(4)^2(-4 \times 10^{-9}) \quad \Rightarrow \quad D_r = \frac{16 \times 10^{-9}}{r^2}$$

So $D_r(r = 5) = \underline{6.4 \times 10^{-10}\,\mathrm{C/m^2}}$.

b) Determine ρ_{s0} such that $\mathbf{D} = 0$ at $r = 7$ m. Since fields will decrease as $1/r^2$, the question could be re-phrased to ask for ρ_{s0} such that $\mathbf{D} = 0$ at *all* points where $r > 6$ m. In this region, the total field will be

$$D_r(r > 6) = \frac{16 \times 10^{-9}}{r^2} + \frac{\rho_{s0}(6)^2}{r^2}$$

Requiring this to be zero, we find $\rho_{s0} = \underline{-(4/9) \times 10^{-9}\,\mathrm{C/m^2}}$.

Bài 10:

3.15. Volume charge density is located as follows: $\rho_v = 0$ for $\rho < 1$ mm and for $\rho > 2$ mm, $\rho_v = 4\rho\ \mu\mathrm{C/m^3}$ for $1 < \rho < 2$ mm.

a) Calculate the total charge in the region $0 < \rho < \rho_1$, $0 < z < L$, where $1 < \rho_1 < 2$ mm: We find

$$Q = \int_0^L \int_0^{2\pi} \int_{.001}^{\rho_1} 4\rho\, \rho\, d\rho\, d\phi\, dz = \underline{\frac{8\pi L}{3}[\rho_1^3 - 10^{-9}]\ \mu\text{C}}$$

where ρ_1 is in meters.

b) Use Gauss' law to determine D_ρ at $\rho = \rho_1$: Gauss' law states that $2\pi\rho_1 L D_\rho = Q$, where Q is the result of part a. Thus

$$D_\rho(\rho_1) = \underline{\frac{4(\rho_1^3 - 10^{-9})}{3\rho_1}\ \mu\text{C/m}^2}$$

where ρ_1 is in meters.

c) Evaluate D_ρ at $\rho = 0.8$ mm, 1.6 mm, and 2.4 mm: At $\rho = 0.8$ mm, no charge is enclosed by a cylindrical gaussian surface of that radius, so $D_\rho(0.8\text{mm}) = \underline{0}$. At $\rho = 1.6$ mm, we evaluate the part b result at $\rho_1 = 1.6$ to obtain:

$$D_\rho(1.6\text{mm}) = \frac{4[(.0016)^3 - (.0010)^3]}{3(.0016)} = \underline{3.6 \times 10^{-6}\ \mu\text{C/m}^2}$$

At $\rho = 2.4$, we evaluate the charge integral of part a from .001 to .002, and Gauss' law is written as

$$2\pi\rho L D_\rho = \frac{8\pi L}{3}[(.002)^2 - (.001)^2]\ \mu\text{C}$$

from which $D_\rho(2.4\text{mm}) = \underline{3.9 \times 10^{-6}\ \mu\text{C/m}^2}$.

Bài 11:

3.17. A cube is defined by $1 < x, y, z < 1.2$. If $\mathbf{D} = 2x^2y\mathbf{a}_x + 3x^2y^2\mathbf{a}_y$ C/m^2:

a) apply Gauss' law to find the total flux leaving the closed surface of the cube. We call the surfaces at $x = 1.2$ and $x = 1$ the front and back surfaces respectively, those at $y = 1.2$ and $y = 1$ the right and left surfaces, and those at $z = 1.2$ and $z = 1$ the top and bottom surfaces. To evaluate the total charge, we integrate $\mathbf{D}\cdot\mathbf{n}$ over all six surfaces and sum the results. We note that there is no z component of $\mathbf{D}$, so there will be no outward flux contributions from the top and bottom surfaces. The fluxes through the remaining four are

$$\Phi = Q = \oint \mathbf{D}\cdot\mathbf{n}\,da = \underbrace{\int_1^{1.2}\int_1^{1.2} 2(1.2)^2 y\,dy\,dz}_{\text{front}} + \underbrace{\int_1^{1.2}\int_1^{1.2} -2(1)^2 y\,dy\,dz}_{\text{back}}$$

$$+ \underbrace{\int_1^{1.2}\int_1^{1.2} -3x^2(1)^2\,dx\,dz}_{\text{left}} + \underbrace{\int_1^{1.2}\int_1^{1.2} 3x^2(1.2)^2\,dx\,dz}_{\text{right}} = \underline{0.1028\text{ C}}$$

b) evaluate $\nabla\cdot\mathbf{D}$ at the center of the cube: This is

$$\nabla\cdot\mathbf{D} = \left[4xy + 6x^2y\right]_{(1.1,1.1)} = 4(1.1)^2 + 6(1.1)^3 = \underline{12.83}$$

Bài 12:

3.21. Calculate the divergence of $\mathbf{D}$ at the point specified if

a) $\mathbf{D} = (1/z^2)\left[10xyz\,\mathbf{a}_x + 5x^2z\,\mathbf{a}_y + (2z^3 - 5x^2y)\,\mathbf{a}_z\right]$ at $P(-2,3,5)$: We find

$$\nabla\cdot\mathbf{D} = \left[\frac{10y}{z} + 0 + 2 + \frac{10x^2y}{z^3}\right]_{(-2,3,5)} = \underline{8.96}$$

b) $\mathbf{D} = 5z^2\mathbf{a}_\rho + 10\rho z\,\mathbf{a}_z$ at $P(3,-45°,5)$: In cylindrical coordinates, we have

$$\nabla\cdot\mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z} = \left[\frac{5z^2}{\rho} + 10\rho\right]_{(3,-45°,5)} = \underline{71.67}$$

c) $\mathbf{D} = 2r\sin\theta\sin\phi\,\mathbf{a}_r + r\cos\theta\sin\phi\,\mathbf{a}_\theta + r\cos\phi\,\mathbf{a}_\phi$ at $P(3,45°,-45°)$: In spherical coordinates, we have

$$\nabla\cdot\mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$= \left[6\sin\theta\sin\phi + \frac{\cos 2\theta\sin\phi}{\sin\theta} - \frac{\sin\phi}{\sin\theta}\right]_{(3,45°,-45°)} = \underline{-2}$$

Bài 13:

3.23. a) A point charge Q lies at the origin. Show that div $\mathbf{D}$ is zero everywhere except at the origin. For a point charge at the origin we know that $\mathbf{D} = Q/(4\pi r^2)\,\mathbf{a}_r$. Using the formula for divergence in spherical coordinates (see problem 3.21 solution), we find in this case that

$$\nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{Q}{4\pi r^2}\right) = 0$$

The above is true provided $r > 0$. When $r = 0$, we have a singularity in $\mathbf{D}$, so its divergence is not defined.

b) Replace the point charge with a uniform volume charge density ρ_{v0} for $0 < r < a$. Relate ρ_{v0} to Q and a so that the total charge is the same. Find div $\mathbf{D}$ everywhere: To achieve the same net charge, we require that $(4/3)\pi a^3 \rho_{v0} = Q$, so $\underline{\rho_{v0} = 3Q/(4\pi a^3)\ \text{C/m}^3}$. Gauss' law tells us that inside the charged sphere

$$4\pi r^2 D_r = \frac{4}{3}\pi r^3 \rho_{v0} = \frac{Qr^3}{a^3}$$

Thus

$$D_r = \frac{Qr}{4\pi a^3}\ \text{C/m}^2 \text{ and } \nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{d}{dr}\left(\frac{Qr^3}{4\pi a^3}\right) = \frac{3Q}{4\pi a^3}$$

as expected. Outside the charged sphere, $\mathbf{D} = Q/(4\pi r^2)\,\mathbf{a}_r$ as before, and the divergence is zero.

Bài 14:

3.25. Within the spherical shell, $3 < r < 4$ m, the electric flux density is given as

$$\mathbf{D} = 5(r-3)^3\,\mathbf{a}_r\ \text{C/m}^2$$

a) What is the volume charge density at $r = 4$? In this case we have

$$\rho_v = \nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{d}{dr}(r^2 D_r) = \frac{5}{r}(r-3)^2(5r-6)\ \text{C/m}^3$$

which we evaluate at $r = 4$ to find $\rho_v(r = 4) = \underline{17.50\ \text{C/m}^3}$.

b) What is the electric flux density at $r = 4$? Substitute $r = 4$ into the given expression to find $\mathbf{D}(4) = \underline{5\,\mathbf{a}_r\ \text{C/m}^2}$

c) How much electric flux leaves the sphere $r = 4$? Using the result of part b, this will be $\Phi = 4\pi(4)^2(5) = \underline{320\pi\ \text{C}}$

d) How much charge is contained within the sphere, $r = 4$? From Gauss' law, this will be the same as the outward flux, or again, $Q = \underline{320\pi\ \text{C}}$.

Bài 15:

3.27. Let $\mathbf{D} = 5.00r^2\mathbf{a}_r$ mC/m^2 for $r \leq 0.08$ m and $\mathbf{D} = 0.205\,\mathbf{a}_r/r^2$ μC/m^2 for $r \geq 0.08$ m (note error in problem statement).

a) Find ρ_v for $r = 0.06$ m: This radius lies within the first region, and so

$$\rho_v = \nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{d}{dr}(r^2 D_r) = \frac{1}{r^2}\frac{d}{dr}(5.00r^4) = 20r \text{ mC/m}^3$$

which when evaluated at $r = 0.06$ yields $\rho_v(r = .06) = \underline{1.20 \text{ mC/m}^3}$.

b) Find ρ_v for $r = 0.1$ m: This is in the region where the second field expression is valid. The $1/r^2$ dependence of this field yields a zero divergence (shown in Problem 3.23), and so the volume charge density is zero at 0.1 m.

c) What surface charge density could be located at $r = 0.08$ m to cause $\mathbf{D} = 0$ for $r > 0.08$ m? The total surface charge should be equal and opposite to the total volume charge. The latter is

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{.08} 20r(\text{mC/m}^3)\, r^2 \sin\theta\, dr\, d\theta\, d\phi = 2.57 \times 10^{-3} \text{ mC} = 2.57\,\mu\text{C}$$

So now

$$\rho_s = -\left[\frac{2.57}{4\pi(.08)^2}\right] = \underline{-32\,\mu\text{C/m}^2}$$

Bài 16:

3.29. In the region of free space that includes the volume $2 < x, y, z < 3$,

$$\mathbf{D} = \frac{2}{z^2}(yz\,\mathbf{a}_x + xz\,\mathbf{a}_y - 2xy\,\mathbf{a}_z) \text{ C/m}^2$$

a) Evaluate the volume integral side of the divergence theorem for the volume defined above: In cartesian, we find $\nabla \cdot \mathbf{D} = 8xy/z^3$. The volume integral side is now

$$\int_{vol} \nabla \cdot \mathbf{D}\, dv = \int_2^3\int_2^3\int_2^3 \frac{8xy}{z^3}\, dxdydz = (9-4)(9-4)\left(\frac{1}{4} - \frac{1}{9}\right) = \underline{3.47 \text{ C}}$$

b. Evaluate the surface integral side for the corresponding closed surface: We call the surfaces at $x = 3$ and $x = 2$ the front and back surfaces respectively, those at $y = 3$ and $y = 2$ the right and left surfaces, and those at $z = 3$ and $z = 2$ the top and bottom surfaces. To evaluate the surface integral side, we integrate $\mathbf{D} \cdot \mathbf{n}$ over all six surfaces and sum the results. Note that since the x component of $\mathbf{D}$ does not vary with x, the outward fluxes from the front and back surfaces will cancel each other. The same is true for the left and right surfaces, since D_y does not vary with y. This leaves only the top and bottom surfaces, where the fluxes are:

$$\oint \mathbf{D} \cdot d\mathbf{S} = \underbrace{\int_2^3\int_2^3 \frac{-4xy}{3^2}\, dxdy}_{\text{top}} - \underbrace{\int_2^3\int_2^3 \frac{-4xy}{2^2}\, dxdy}_{\text{bottom}} = (9-4)(9-4)\left(\frac{1}{4} - \frac{1}{9}\right) = \underline{3.47 \text{ C}}$$

Bài 17:

3.31. Given the flux density

$$\mathbf{D} = \frac{16}{r}\cos(2\theta)\,\mathbf{a}_\theta \text{ C/m}^2,$$

use two different methods to find the total charge within the region $1 < r < 2$ m, $1 < \theta < 2$ rad, $1 < \phi < 2$ rad: We use the divergence theorem and first evaluate the surface integral side. We are evaluating the net outward flux through a curvilinear "cube", whose boundaries are defined by the specified ranges. The flux contributions will be only through the surfaces of constant θ, however, since $\mathbf{D}$ has only a θ component. On a constant-theta surface, the differential area is $da = r\sin\theta dr d\phi$, where θ is fixed at the surface location. Our flux integral becomes

$$\oint \mathbf{D}\cdot d\mathbf{S} = -\underbrace{\int_1^2\int_1^2 \frac{16}{r}\cos(2)\,r\sin(1)\,dr d\phi}_{\theta=1} + \underbrace{\int_1^2\int_1^2 \frac{16}{r}\cos(4)\,r\sin(2)\,dr d\phi}_{\theta=2}$$

$$= -16\left[\cos(2)\sin(1) - \cos(4)\sin(2)\right] = \underline{-3.91 \text{ C}}$$

We next evaluate the volume integral side of the divergence theorem, where in this case,

$$\nabla\cdot\mathbf{D} = \frac{1}{r\sin\theta}\frac{d}{d\theta}(\sin\theta\, D_\theta) = \frac{1}{r\sin\theta}\frac{d}{d\theta}\left[\frac{16}{r}\cos 2\theta\sin\theta\right] = \frac{16}{r^2}\left[\frac{\cos 2\theta\cos\theta}{\sin\theta} - 2\sin 2\theta\right]$$

We now evaluate:

$$\int_{vol}\nabla\cdot\mathbf{D}\,dv = \int_1^2\int_1^2\int_1^2 \frac{16}{r^2}\left[\frac{\cos 2\theta\cos\theta}{\sin\theta} - 2\sin 2\theta\right] r^2\sin\theta\, dr d\theta d\phi$$

The integral simplifies to

$$\int_1^2\int_1^2\int_1^2 16[\cos 2\theta\cos\theta \quad 2\sin 2\theta\sin\theta]\,dr d\theta d\phi = 8\int_1^2 [3\cos 3\theta \quad \cos\theta]\,d\theta = \underline{\quad 3.91 \text{ C}}$$

Chương 4: Năng lượng – Điện thế

Bài 1:

4.1. The value of $\mathbf{E}$ at $P(\rho = 2,\ \phi = 40°,\ z = 3)$ is given as $\mathbf{E} = 100\mathbf{a}_\rho - 200\mathbf{a}_\phi + 300\mathbf{a}_z$ V/m. Determine the incremental work required to move a $20\,\mu$C charge a distance of 6 μm:

a) in the direction of $\mathbf{a}_\rho$: The incremental work is given by $dW = -q\,\mathbf{E}\cdot d\mathbf{L}$, where in this case, $d\mathbf{L} = d\rho\,\mathbf{a}_\rho = 6\times10^{-6}\,\mathbf{a}_\rho$. Thus

$$dW = -(20\times10^{-6}\,\text{C})(100\,\text{V/m})(6\times10^{-6}\,\text{m}) = -12\times10^{-9}\ \text{J} = \underline{-12\,\text{nJ}}$$

b) in the direction of $\mathbf{a}_\phi$: In this case $d\mathbf{L} = 2\,d\phi\,\mathbf{a}_\phi = 6\times10^{-6}\,\mathbf{a}_\phi$, and so

$$dW = -(20\times10^{-6})(-200)(6\times10^{-6}) = 2.4\times10^{-8}\,\text{J} = \underline{24\ \text{nJ}}$$

c) in the direction of $\mathbf{a}_z$: Here, $d\mathbf{L} = dz\,\mathbf{a}_z = 6\times10^{-6}\,\mathbf{a}_z$, and so

$$dW = -(20\times10^{-6})(300)(6\times10^{-6}) = -3.6\times10^{-8}\,\text{J} = \underline{-36\ \text{nJ}}$$

d) in the direction of $\mathbf{E}$: Here, $d\mathbf{L} = 6\times10^{-6}\,\mathbf{a}_E$, where

$$\mathbf{a}_E = \frac{100\mathbf{a}_\rho - 200\mathbf{a}_\phi + 300\mathbf{a}_z}{[100^2+200^2+300^2]^{1/2}} = 0.267\,\mathbf{a}_\rho - 0.535\,\mathbf{a}_\phi + 0.802\,\mathbf{a}_z$$

Thus

$$dW = -(20\times10^{-6})[100\mathbf{a}_\rho - 200\mathbf{a}_\phi + 300\mathbf{a}_z]\cdot[0.267\,\mathbf{a}_\rho - 0.535\,\mathbf{a}_\phi + 0.802\,\mathbf{a}_z](6\times10^{-6})$$
$$= \underline{-44.9\,\text{nJ}}$$

e) In the direction of $\mathbf{G} = 2\,\mathbf{a}_x - 3\,\mathbf{a}_y + 4\,\mathbf{a}_z$: In this case, $d\mathbf{L} = 6\times10^{-6}\,\mathbf{a}_G$, where

$$\mathbf{a}_G = \frac{2\mathbf{a}_x - 3\mathbf{a}_y + 4\mathbf{a}_z}{[2^2+3^2+4^2]^{1/2}} = 0.371\,\mathbf{a}_x - 0.557\,\mathbf{a}_y + 0.743\,\mathbf{a}_z$$

So now

$$dW = -(20\times10^{-6})[100\mathbf{a}_\rho - 200\mathbf{a}_\phi + 300\mathbf{a}_z]\cdot[0.371\,\mathbf{a}_x - 0.557\,\mathbf{a}_y + 0.743\,\mathbf{a}_z](6\times10^{-6})$$
$$= -(20\times10^{-6})\,[37.1(\mathbf{a}_\rho\cdot\mathbf{a}_x) - 55.7(\mathbf{a}_\rho\cdot\mathbf{a}_y) - 74.2(\mathbf{a}_\phi\cdot\mathbf{a}_x) + 111.4(\mathbf{a}_\phi\cdot\mathbf{a}_y)$$
$$+\ 222.9]\,(6\times10^{-6})$$

where, at P, $(\mathbf{a}_\rho\cdot\mathbf{a}_x) = (\mathbf{a}_\phi\cdot\mathbf{a}_y) = \cos(40°) = 0.766$, $(\mathbf{a}_\rho\cdot\mathbf{a}_y) = \sin(40°) = 0.643$, and $(\mathbf{a}_\phi\cdot\mathbf{a}_x) = -\sin(40°) = -0.643$. Substituting these results in

$$dW = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6}) = \underline{-41.8\,\text{nJ}}$$

Bài 2:

4.3. If $\mathbf{E} = 120\,\mathbf{a}_\rho$ V/m, find the incremental amount of work done in moving a $50\,\mu$m charge a distance of 2 mm from:

a) $P(1,2,3)$ toward $Q(2,1,4)$: The vector along this direction will be $Q - P = (1,-1,1)$ from which $\mathbf{a}_{PQ} = [\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z]/\sqrt{3}$. We now write

$$dW = -q\mathbf{E}\cdot d\mathbf{L} = -(50\times 10^{-6})\left[120\mathbf{a}_\rho \cdot \frac{(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z}{\sqrt{3}}\right](2\times 10^{-3})$$
$$= -(50\times 10^{-6})(120)\left[(\mathbf{a}_\rho\cdot\mathbf{a}_x) - (\mathbf{a}_\rho\cdot\mathbf{a}_y)\right]\frac{1}{\sqrt{3}}(2\times 10^{-3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4^\circ$. Thus $(\mathbf{a}_\rho\cdot\mathbf{a}_x) = \cos(63.4) = 0.447$ and $(\mathbf{a}_\rho\cdot\mathbf{a}_y) = \sin(63.4) = 0.894$. Substituting these, we obtain $dW = \underline{3.1\,\mu\text{J}}$.

b) $Q(2,1,4)$ toward $P(1,2,3)$: A little thought is in order here: Note that the field has only a radial component and does not depend on ϕ or z. Note also that P and Q are at the same radius ($\sqrt{5}$) from the z axis, but have different ϕ and z coordinates. We could just as well position the two points at the same z location and the problem would not change. If this were so, then moving along a straight line between P and Q would thus involve moving along a chord of a circle whose radius is $\sqrt{5}$. Halfway along this line is a point of symmetry in the field (make a sketch to see this). This means that when starting from either point, the initial force will be the same. Thus the answer is $dW = \underline{3.1\,\mu\text{J}}$ as in part a. This is also found by going through the same procedure as in part a, but with the direction (roles of P and Q) reversed.

Bài 3:

4.11. Let a uniform surface charge density of $5\,\text{nC/m}^2$ be present at the $z = 0$ plane, a uniform line charge density of $8\,\text{nC/m}$ be located at $x = 0$, $z = 4$, and a point charge of $2\,\mu\text{C}$ be present at $P(2, 0, 0)$. If $V = 0$ at $M(0, 0, 5)$, find V at $N(1, 2, 3)$: We need to find a potential function for the combined charges which is zero at M. That for the point charge we know to be

$$V_p(r) = \frac{Q}{4\pi\epsilon_0 r}$$

Potential functions for the sheet and line charges can be found by taking indefinite integrals of the electric fields for those distributions. For the line charge, we have

$$V_l(\rho) = -\int \frac{\rho_l}{2\pi\epsilon_0 \rho} d\rho + C_1 = -\frac{\rho_l}{2\pi\epsilon_0} \ln(\rho) + C_1$$

For the sheet charge, we have

$$V_s(z) = -\int \frac{\rho_s}{2\epsilon_0} dz + C_2 = -\frac{\rho_s}{2\epsilon_0} z + C_2$$

The total potential function will be the sum of the three. Combining the integration constants, we obtain:

$$V = \frac{Q}{4\pi\epsilon_0 r} - \frac{\rho_l}{2\pi\epsilon_0} \ln(\rho) - \frac{\rho_s}{2\epsilon_0} z + C$$

The terms in this expression are not referenced to a common origin, since the charges are at different positions. The parameters r, ρ, and z are *scalar distances* from the charges, and will be treated as such here. To evaluate the constant, C, we first look at point M, where $V_T = 0$. At M, $r = \sqrt{2^2 + 5^2} = \sqrt{29}$, $\rho = 1$, and $z = 5$. We thus have

$$0 = \frac{2 \times 10^{-6}}{4\pi\epsilon_0 \sqrt{29}} - \frac{8 \times 10^{-9}}{2\pi\epsilon_0} \ln(1) - \frac{5 \times 10^{-9}}{2\epsilon_0} 5 + C \quad \Rightarrow \quad C = -1.93 \times 10^3 \text{ V}$$

At point N, $r = \sqrt{1 + 4 + 9} = \sqrt{14}$, $\rho = \sqrt{2}$, and $z = 3$. The potential at N is thus

$$V_N = \frac{2 \times 10^{-6}}{4\pi\epsilon_0 \sqrt{14}} - \frac{8 \times 10^{-9}}{2\pi\epsilon_0} \ln(\sqrt{2}) - \frac{5 \times 10^{-9}}{2\epsilon_0} (3) - 1.93 \times 10^3 = 1.98 \times 10^3 \text{ V} = \underline{1.98\,\text{kV}}$$

Bài 5:

4.15. Two uniform line charges, 8 nC/m each, are located at $x = 1$, $z = 2$, and at $x = -1$, $y = 2$ in free space. If the potential at the origin is 100 V, find V at $P(4, 1, 3)$: The net potential function for the two charges would in general be:

$$V = -\frac{\rho_l}{2\pi\epsilon_0}\ln(R_1) - \frac{\rho_l}{2\pi\epsilon_0}\ln(R_2) + C$$

At the origin, $R_1 = R_2 = \sqrt{5}$, and $V = 100$ V. Thus, with $\rho_l = 8 \times 10^{-9}$,

$$100 = -2\frac{(8 \times 10^{-9})}{2\pi\epsilon_0}\ln(\sqrt{5}) + C \quad \Rightarrow \quad C = 331.6 \text{ V}$$

At $P(4, 1, 3)$, $R_1 = |(4, 1, 3) - (1, 1, 2)| = \sqrt{10}$ and $R_2 = |(4, 1, 3) - (-1, 2, 3)| = \sqrt{26}$. Therefore

$$V_P = -\frac{(8 \times 10^{-9})}{2\pi\epsilon_0}\left[\ln(\sqrt{10}) + \ln(\sqrt{26})\right] + 331.6 = \underline{-68.4 \text{ V}}$$

Bài 6:

Uniform surface charge densities of 6 and 2 nC/m^2 are present at $\rho = 2$ and 6 cm respectively, in free space. Assume $V = 0$ at $\rho = 4$ cm, and calculate V at:

a) $\rho = 5$ cm: Since $V = 0$ at 4 cm, the potential at 5 cm will be the potential difference between points 5 and 4:

$$V_5 = -\int_4^5 \mathbf{E} \cdot d\mathbf{L} = -\int_4^5 \frac{a\rho_{sa}}{\epsilon_0 \rho}\, d\rho = -\frac{(.02)(6 \times 10^{-9})}{\epsilon_0}\ln\left(\frac{5}{4}\right) = \underline{-3.026 \text{ V}}$$

b) $\rho = 7$ cm: Here we integrate piecewise from $\rho = 4$ to $\rho = 7$:

$$V_7 = -\int_4^6 \frac{a\rho_{sa}}{\epsilon_0 \rho} d\rho - \int_6^7 \frac{(a\rho_{sa} + b\rho_{sb})}{\epsilon_0 \rho} d\rho$$

With the given values, this becomes

$$V_7 = -\left[\frac{(.02)(6 \times 10^{-9})}{\epsilon_0}\right]\ln\left(\frac{6}{4}\right) - \left[\frac{(.02)(6 \times 10^{-9}) + (.06)(2 \times 10^{-9})}{\epsilon_0}\right]\ln\left(\frac{7}{6}\right)$$

$$= \underline{-9.678 \text{ V}}$$

Bài 7:

4.19. The annular surface, 1 cm $< \rho <$ 3 cm, $z = 0$, carries the nonuniform surface charge density $\rho_s = 5\rho\,\text{nC/m}^2$. Find V at $P(0, 0, 2\,\text{cm})$ if $V = 0$ at infinity: We use the superposition integral form:

$$V_P = \int\int \frac{\rho_s\,da}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|}$$

where $\mathbf{r} = z\mathbf{a}_z$ and $\mathbf{r}' = \rho\mathbf{a}_\rho$. We integrate over the surface of the annular region, with $da = \rho\,d\rho\,d\phi$. Substituting the given values, we find

$$V_P = \int_0^{2\pi}\int_{.01}^{.03} \frac{(5 \times 10^{-9})\rho^2\,d\rho\,d\phi}{4\pi\epsilon_0\sqrt{\rho^2 + z^2}}$$

Substituting $z = .02$, and using tables, the integral evaluates as

$$V_P = \left[\frac{(5 \times 10^{-9})}{2\epsilon_0}\right]\left[\frac{\rho}{2}\sqrt{\rho^2 + (.02)^2} - \frac{(.02)^2}{2}\ln(\rho + \sqrt{\rho^2 + (.02)^2})\right]_{.01}^{.03} = \underline{.081\,\text{V}}$$

Bài 8:

4.23. It is known that the potential is given as $V = 80\rho^{.6}$ V. Assuming free space conditions, find:

a) $\mathbf{E}$: We find this through

$$\mathbf{E} = -\nabla V = -\frac{dV}{d\rho}\mathbf{a}_\rho = \underline{-48\rho^{-.4}\,\text{V/m}}$$

b) the volume charge density at $\rho = .5$ m: Using $\mathbf{D} = \epsilon_0\mathbf{E}$, we find the charge density through

$$\rho_v\Big|_{.5} = [\nabla \cdot \mathbf{D}]_{.5} = \left(\frac{1}{\rho}\right)\frac{d}{d\rho}(\rho D_\rho)\Big|_{.5} = -28.8\epsilon_0\rho^{-1.4}\Big|_{.5} = \underline{-673\,\text{pC/m}^3}$$

c) the total charge lying within the closed surface $\rho = .6$, $0 < z < 1$: The easiest way to do this calculation is to evaluate D_ρ at $\rho = .6$ (noting that it is constant), and then multiply by the cylinder area: Using part a, we have $D_\rho\Big|_{.6} = -48\epsilon_0(.6)^{-.4} = -521\,\text{pC/m}^2$. Thus $Q = -2\pi(.6)(1)521 \times 10^{-12}\,\text{C} = \underline{-1.96\,\text{nC}}$.

Bài 9:

4.25. Within the cylinder $\rho = 2$, $0 < z < 1$, the potential is given by $V = 100 + 50\rho + 150\rho \sin\phi$ V.

a) Find V, $\mathbf{E}$, $\mathbf{D}$, and ρ_v at $P(1, 60°, 0.5)$ in free space: First, substituting the given point, we find $V_P = \underline{279.9\,\text{V}}$. Then,

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho}\mathbf{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi = -\left[50 + 150\sin\phi\right]\mathbf{a}_\rho - \left[150\cos\phi\right]\mathbf{a}_\phi$$

Evaluate the above at P to find $\mathbf{E}_P = \underline{-179.9\mathbf{a}_\rho - 75.0\mathbf{a}_\phi \text{ V/m}}$

Now $\mathbf{D} = \epsilon_0 \mathbf{E}$, so $\mathbf{D}_P = \underline{-1.59\mathbf{a}_\rho - .664\mathbf{a}_\phi \text{ nC/m}^2}$. Then

$$\rho_v = \nabla \cdot \mathbf{D} = \left(\frac{1}{\rho}\right)\frac{d}{d\rho}\left(\rho D_\rho\right) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} = \left[-\frac{1}{\rho}(50 + 150\sin\phi) + \frac{1}{\rho}150\sin\phi\right]\epsilon_0 = -\frac{50}{\rho}\epsilon_0 \text{ C}$$

At P, this is $\rho_{vP} = \underline{-443 \text{ pC/m}^3}$.

b) How much charge lies within the cylinder? We will integrate ρ_v over the volume to obtain:

$$Q = \int_0^1 \int_0^{2\pi} \int_0^2 -\frac{50\epsilon_0}{\rho}\rho\, d\rho\, d\phi\, dz = -2\pi(50)\epsilon_0(2) = \underline{-5.56\,\text{nC}}$$

Bài 10:

4.27. Two point charges, 1 nC at $(0, 0, 0.1)$ and -1 nC at $(0, 0, -0.1)$, are in free space.

a) Calculate V at $P(0.3, 0, 0.4)$: Use

$$V_P = \frac{q}{4\pi\epsilon_0|\mathbf{R}^+|} - \frac{q}{4\pi\epsilon_0|\mathbf{R}^-|}$$

where $\mathbf{R}^+ = (.3, 0, .3)$ and $\mathbf{R}^- = (.3, 0, .5)$, so that $|\mathbf{R}^+| = 0.424$ and $|\mathbf{R}^-| = 0.583$. Thus

$$V_P = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{1}{.424} - \frac{1}{.583}\right] = \underline{5.78\text{ V}}$$

b) Calculate $|\mathbf{E}|$ at P: Use

$$\mathbf{E}_P = \frac{q(.3\mathbf{a}_x + .3\mathbf{a}_z)}{4\pi\epsilon_0(.424)^3} - \frac{q(.3\mathbf{a}_x + .5\mathbf{a}_z)}{4\pi\epsilon_0(.583)^3} = \frac{10^{-9}}{4\pi\epsilon_0}\left[2.42\mathbf{a}_x + 1.41\mathbf{a}_z\right] \text{ V/m}$$

Taking the magnitude of the above, we find $|\mathbf{E}_P| = \underline{25.2\,\text{V/m}}$.

c) Now treat the two charges as a dipole at the origin and find V at P: In spherical coordinates, P is located at $r = \sqrt{.3^2 + .4^2} = .5$ and $\theta = \sin^{-1}(.3/.5) = 36.9°$. Assuming a dipole in far-field, we have

$$V_P = \frac{qd\cos\theta}{4\pi\epsilon_0 r^2} = \frac{10^{-9}(.2)\cos(36.9°)}{4\pi\epsilon_0(.5)^2} = \underline{5.76\text{ V}}$$

Bài 11:

4.31. A potential field in free space is expressed as $V = 20/(xyz)$ V.

a) Find the total energy stored within the cube $1 < x, y, z < 2$. We integrate the energy density over the cube volume, where $w_E = (1/2)\epsilon_0 \mathbf{E} \cdot \mathbf{E}$, and where

$$\mathbf{E} = -\nabla V = 20\left[\frac{1}{x^2yz}\mathbf{a}_x + \frac{1}{xy^2z}\mathbf{a}_y + \frac{1}{xyz^2}\mathbf{a}_z\right] \text{ V/m}$$

The energy is now

$$W_E = 200\epsilon_0 \int_1^2\int_1^2\int_1^2 \left[\frac{1}{x^4y^2z^2} + \frac{1}{x^2y^4z^2} + \frac{1}{x^2y^2z^4}\right] dx\,dy\,dz$$

The integral evaluates as follows:

$$\begin{aligned} W_E &= 200\epsilon_0 \int_1^2\int_1^2 \left[-\left(\frac{1}{3}\right)\frac{1}{x^3y^2z^2} - \frac{1}{xy^4z^2} - \frac{1}{xy^2z^4}\right]_1^2 dy\,dz \\ &= 200\epsilon_0 \int_1^2\int_1^2 \left[\left(\frac{7}{24}\right)\frac{1}{y^2z^2} + \left(\frac{1}{2}\right)\frac{1}{y^4z^2} + \left(\frac{1}{2}\right)\frac{1}{y^2z^4}\right] dy\,dz \\ &= 200\epsilon_0 \int_1^2 \left[-\left(\frac{7}{24}\right)\frac{1}{yz^2} - \left(\frac{1}{6}\right)\frac{1}{y^3z^2} - \left(\frac{1}{2}\right)\frac{1}{yz^4}\right]_1^2 dz \\ &= 200\epsilon_0 \int_1^2 \left[\left(\frac{7}{48}\right)\frac{1}{z^2} + \left(\frac{7}{48}\right)\frac{1}{z^2} + \left(\frac{1}{4}\right)\frac{1}{z^4}\right] dz \\ &= 200\epsilon_0(3)\left[\frac{7}{96}\right] = \underline{387\text{ pJ}} \end{aligned}$$

b) What value would be obtained by assuming a uniform energy density equal to the value at the center of the cube? At $C(1.5, 1.5, 1.5)$ the energy density is

$$w_E = 200\epsilon_0(3)\left[\frac{1}{(1.5)^4(1.5)^2(1.5)^2}\right] = 2.07 \times 10^{-10} \text{ J/m}^3$$

This, multiplied by a cube volume of 1, produces an energy value of $\underline{207\text{ pJ}}$.

Bài 12:

4.33. A copper sphere of radius 4 cm carries a uniformly-distributed total charge of 5 μC in free space.

a) Use Gauss' law to find **D** external to the sphere: with a spherical Gaussian surface at radius r, D will be the total charge divided by the area of this sphere, and will be $\mathbf{a}_r$-directed. Thus

$$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r = \frac{5\times 10^{-6}}{4\pi r^2}\mathbf{a}_r \text{ C/m}^2$$

4.33b) Calculate the total energy stored in the electrostatic field: Use

$$W_E = \int_{vol} \frac{1}{2}\mathbf{D}\cdot\mathbf{E}\,dv = \int_0^{2\pi}\int_0^{\pi}\int_{.04}^{\infty} \frac{1}{2}\frac{(5\times 10^{-6})^2}{16\pi^2\epsilon_0 r^4}\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= (4\pi)\left(\frac{1}{2}\right)\frac{(5\times 10^{-6})^2}{16\pi^2\epsilon_0}\int_{.04}^{\infty}\frac{dr}{r^2} = \frac{25\times 10^{-12}}{8\pi\epsilon_0}\frac{1}{.04} = \underline{2.81\text{ J}}$$

c) Use $W_E = Q^2/(2C)$ to calculate the capacitance of the isolated sphere: We have

$$C = \frac{Q^2}{2W_E} = \frac{(5\times 10^{-6})^2}{2(2.81)} = 4.45\times 10^{-12}\text{ F} = \underline{4.45\text{ pF}}$$

Bài 13:

4.35. Four 0.8 nC point charges are located in free space at the corners of a square 4 cm on a side.

a) Find the total potential energy stored: This will be given by

$$W_E = \frac{1}{2}\sum_{n=1}^{4} q_n V_n$$

where V_n in this case is the potential at the location of any one of the point charges that arises from the other three. This will be (for charge 1)

$$V_1 = V_{21} + V_{31} + V_{41} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{.04} + \frac{1}{.04} + \frac{1}{.04\sqrt{2}}\right]$$

Taking the summation produces a factor of 4, since the situation is the same at all four points. Consequently,

$$W_E = \frac{1}{2}(4)q_1V_1 = \frac{(.8\times10^{-9})^2}{2\pi\epsilon_0(.04)}\left[2 + \frac{1}{\sqrt{2}}\right] = 7.79\times10^{-7}\,\text{J} = \underline{0.779\,\mu\text{J}}$$

b) A fifth 0.8 nC charge is installed at the center of the square. Again find the total stored energy: This will be the energy found in part *a* plus the amount of work done in moving the fifth charge into position from infinity. The latter is just the potential at the square center arising from the original four charges, times the new charge value, or

$$\Delta W_E = \frac{4(.8\times10^{-9})^2}{4\pi\epsilon_0(.04\sqrt{2}/2)} = .813\,\mu\text{J}$$

The total energy is now

$$W_{E\,net} = W_E(\text{part a}) + \Delta W_E = .779 + .813 = \underline{1.59\,\mu\text{J}}$$

Chương 5: Vật dẫn – Điện môi – Điện dung

Bài 1:

5.1. Given the current density $\mathbf{J} = -10^4[\sin(2x)e^{-2y}\mathbf{a}_x + \cos(2x)e^{-2y}\mathbf{a}_y]$ kA/m^2:

a) Find the total current crossing the plane $y = 1$ in the $\mathbf{a}_y$ direction in the region $0 < x < 1$, $0 < z < 2$: This is found through

$$I = \int\int_S \mathbf{J}\cdot\mathbf{n}\Big|_S da = \int_0^2\int_0^1 \mathbf{J}\cdot\mathbf{a}_y\Big|_{y=1} dx\,dz = \int_0^2\int_0^1 -10^4\cos(2x)e^{-2}\,dx\,dz$$
$$= -10^4(2)\frac{1}{2}\sin(2x)\Big|_0^1 e^{-2} = \underline{-1.23\,\text{MA}}$$

b) Find the total current leaving the region $0 < x, x < 1$, $2 < z < 3$ by integrating $\mathbf{J}\cdot\mathbf{dS}$ over the surface of the cube: Note first that current through the top and bottom surfaces will not exist, since $\mathbf{J}$ has no z component. Also note that there will be no current through the $x = 0$ plane, since $J_x = 0$ there. Current will pass through the three remaining surfaces, and will be found through

$$I = \int_2^3\int_0^1 \mathbf{J}\cdot(-\mathbf{a}_y)\Big|_{y=0} dx\,dz + \int_2^3\int_0^1 \mathbf{J}\cdot(\mathbf{a}_y)\Big|_{y=1} dx\,dz + \int_2^3\int_0^1 \mathbf{J}\cdot(\mathbf{a}_x)\Big|_{x=1} dy\,dz$$
$$= 10^4\int_2^3\int_0^1\left[\cos(2x)e^{-0} - \cos(2x)e^{-2}\right] dx\,dz - 10^4\int_2^3\int_0^1 \sin(2)e^{-2y}\,dy\,dz$$
$$= 10^4\left(\frac{1}{2}\right)\sin(2x)\Big|_0^1(3-2)\left[1 - e^{-2}\right] + 10^4\left(\frac{1}{2}\right)\sin(2)e^{-2y}\Big|_0^1(3-2) = \underline{0}$$

c) Repeat part b, but use the divergence theorem: We find the net outward current through the surface of the cube by integrating the divergence of $\mathbf{J}$ over the cube volume. We have

$$\nabla\cdot\mathbf{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = -10^{-4}\left[2\cos(2x)e^{-2y} - 2\cos(2x)e^{-2y}\right] = \underline{0} \quad \text{as expected}$$

Bài 2:

5.3. Let

$$\mathbf{J} = \frac{400 \sin\theta}{r^2 + 4}\, \mathbf{a}_r \ \mathrm{A/m^2}$$

a) Find the total current flowing through that portion of the spherical surface $r = 0.8$, bounded by $0.1\pi < \theta < 0.3\pi$, $0 < \phi < 2\pi$: This will be

$$I = \int\int \mathbf{J}\cdot\mathbf{n}\Big|_S da = \int_0^{2\pi}\int_{.1\pi}^{.3\pi} \frac{400\sin\theta}{(.8)^2+4}(.8)^2 \sin\theta\, d\theta\, d\phi = \frac{400(.8)^2 2\pi}{4.64}\int_{.1\pi}^{.3\pi} \sin^2 d\theta$$

$$= 346.5 \int_{.1\pi}^{.3\pi} \frac{1}{2}[1-\cos(2\theta)]\, d\theta = \underline{77.4\ \mathrm{A}}$$

b) Find the average value of $\mathbf{J}$ over the defined area. The area is

$$\text{Area} = \int_0^{2\pi}\int_{.1\pi}^{.3\pi} (.8)^2 \sin\theta\, d\theta\, d\phi = 1.46\,\mathrm{m}^2$$

The average current density is thus $\mathbf{J}_{avg} = (77.4/1.46)\,\mathbf{a}_r = \underline{53.0\,\mathbf{a}_r\ \mathrm{A/m^2}}$.

Bài 3:

5.5. Let

$$\mathbf{J} = \frac{25}{\rho}\,\mathbf{a}_\rho - \frac{20}{\rho^2 + 0.01}\,\mathbf{a}_z \ \mathrm{A/m^2}$$

a) Find the total current crossing the plane $z = 0.2$ in the $\mathbf{a}_z$ direction for $\rho < 0.4$: Use

$$I = \int\int_S \mathbf{J}\cdot\mathbf{n}\Big|_{z=.2} da = \int_0^{2\pi}\int_0^{.4} \frac{-20}{\rho^2+.01}\rho\, d\rho\, d\phi$$

$$= -\left(\frac{1}{2}\right) 20\ln(.01+\rho^2)\Big|_0^{.4}(2\pi) = -20\pi\ln(17) = \underline{-178.0\ \mathrm{A}}$$

b) Calculate $\partial\rho_v/\partial t$: This is found using the equation of continuity:

$$\frac{\partial\rho_v}{\partial t} = -\nabla\cdot\mathbf{J} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho J_\rho) + \frac{\partial J_z}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(25) + \frac{\partial}{\partial z}\left(\frac{-20}{\rho^2+.01}\right) = \underline{0}$$

c) Find the outward current crossing the closed surface defined by $\rho = 0.01$, $\rho = 0.4$, $z = 0$, and $z = 0.2$: This will be

$$I = \int_0^{.2}\int_0^{2\pi} \frac{25}{.01}\mathbf{a}_\rho\cdot(-\mathbf{a}_\rho)(.01)\, d\phi\, dz + \int_0^{.2}\int_0^{2\pi} \frac{25}{.4}\mathbf{a}_\rho\cdot(\mathbf{a}_\rho)(.4)\, d\phi\, dz$$

$$+ \int_0^{2\pi}\int_0^{.4} \frac{-20}{\rho^2+.01}\mathbf{a}_z\cdot(-\mathbf{a}_z)\,\rho\, d\rho\, d\phi + \int_0^{2\pi}\int_0^{.4} \frac{-20}{\rho^2+.01}\mathbf{a}_z\cdot(\mathbf{a}_z)\,\rho\, d\rho\, d\phi = \underline{0}$$

Bài 4:

5.9a. Using data tabulated in Appendix C, calculate the required diameter for a 2-m long nichrome wire that will dissipate an average power of 450 W when 120 V rms at 60 Hz is applied to it: The required resistance will be

$$R = \frac{V^2}{P} = \frac{l}{\sigma(\pi a^2)}$$

Thus the diameter will be

$$d = 2a = 2\sqrt{\frac{lP}{\sigma \pi V^2}} = 2\sqrt{\frac{2(450)}{(10^6)\pi(120)^2}} = 2.8 \times 10^{-4}\,\text{m} = \underline{0.28\,\text{mm}}$$

b) Calculate the rms current density in the wire: The rms current will be $I = 450/120 = 3.75\,\text{A}$. Thus

$$J = \frac{3.75}{\pi\left(2.8 \times 10^{-4}/2\right)^2} = \underline{6.0 \times 10^7\,\text{A/m}^2}$$

Bài 5:

5.11. Two perfectly-conducting cylindrical surfaces of length l are located at $\rho = 3$ and $\rho = 5$ cm. The total current passing radially outward through the medium between the cylinders is 3 A dc.

a) Find the voltage and resistance between the cylinders, and **E** in the region between the cylinders, if a conducting material having $\sigma = 0.05\,\text{S/m}$ is present for $3 < \rho < 5$ cm: Given the current, and knowing that it is radially-directed, we find the current density by dividing it by the area of a cylinder of radius ρ and length l:

$$\mathbf{J} = \frac{3}{2\pi\rho l}\,\mathbf{a}_\rho\ \text{A/m}^2$$

Then the electric field is found by dividing this result by σ:

$$\mathbf{E} = \frac{3}{2\pi\sigma\rho l}\,\mathbf{a}_\rho = \underline{\frac{9.55}{\rho l}\mathbf{a}_\rho\ \text{V/m}}$$

The voltage between cylinders is now:

$$V = -\int_5^3 \mathbf{E}\cdot d\mathbf{L} = \int_3^5 \frac{9.55}{\rho l}\mathbf{a}_\rho \cdot \mathbf{a}_\rho d\rho = \frac{9.55}{l}\ln\left(\frac{5}{3}\right) = \underline{\frac{4.88}{l}\ \text{V}}$$

Now, the resistance will be

$$R = \frac{V}{I} = \frac{4.88}{3l} = \underline{\frac{1.63}{l}\ \Omega}$$

Bài 6:

5.15. Let $V = 10(\rho+1)z^2\cos\phi\,\mathrm{V}$ in free space.

a) Let the equipotential surface $V = 20\,\mathrm{V}$ define a conductor surface. Find the equation of the conductor surface: Set the given potential function equal to 20, to find:

$$\underline{(\rho+1)z^2\cos\phi = 2}$$

b) Find ρ and $\mathbf{E}$ at that point on the conductor surface where $\phi = 0.2\pi$ and $z = 1.5$: At the given values of ϕ and z, we solve the equation of the surface found in part a for ρ, obtaining $\rho = \underline{.10}$. Then

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho}\mathbf{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi - \frac{\partial V}{\partial z}\mathbf{a}_z$$
$$= -10z^2\cos\phi\,\mathbf{a}_\rho + 10\,\frac{\rho+1}{\rho}z^2\sin\phi\,\mathbf{a}_\phi - 20(\rho+1)z\cos\phi\,\mathbf{a}_z$$

Then

$$\mathbf{E}(.10, .2\pi, 1.5) = \underline{-18.2\,\mathbf{a}_\rho + 145\,\mathbf{a}_\phi - 26.7\,\mathbf{a}_z\ \mathrm{V/m}}$$

c) Find $|\rho_s|$ at that point: Since $\mathbf{E}$ is at the perfectly-conducting surface, it will be normal to the surface, so we may write:

$$\rho_s = \epsilon_0\mathbf{E}\cdot\mathbf{n}\Big|_{\text{surface}} = \epsilon_0\frac{\mathbf{E}\cdot\mathbf{E}}{|\mathbf{E}|} = \epsilon_0\sqrt{\mathbf{E}\cdot\mathbf{E}} = \epsilon_0\sqrt{(18.2)^2+(145)^2+(26.7)^2} = \underline{1.32\ \mathrm{nC/m^2}}$$

Bài 7:

5.17. Given the potential field $V - 100xz/(x^2+4)$ V. in free space:

a) Find $\mathbf{D}$ at the surface $z = 0$: Use

$$\mathbf{E} = -\nabla V = -100z\frac{\partial}{\partial x}\left(\frac{x}{x^2+4}\right)\mathbf{a}_x - 0\,\mathbf{a}_y - \frac{100x}{x^2+4}\mathbf{a}_z\ \mathrm{V/m}$$

At $z = 0$, we use this to find $\mathbf{D}(z=0) = \epsilon_0\mathbf{E}(z=0) = \underline{-100\epsilon_0 x/(x^2+4)\,\mathbf{a}_z\ \mathrm{C/m^2}}$.

b) Show that the $z = 0$ surface is an equipotential surface: There are two reasons for this: 1) $\mathbf{E}$ at $z = 0$ is everywhere z-directed, and so moving a charge around on the surface involves doing no work; 2) When evaluating the given potential function at $z = 0$, the result is 0 for all x and y.

c) Assume that the $z = 0$ surface is a conductor and find the total charge on that portion of the conductor defined by $0 < x < 2$, $-3 < y < 0$: We have

$$\rho_s = \mathbf{D}\cdot\mathbf{a}_z\Big|_{z=0} = -\frac{100\epsilon_0 x}{x^2+4}\ \mathrm{C/m^2}$$

So

$$Q = \int_{-3}^{0}\int_{0}^{2} -\frac{100\epsilon_0 x}{x^2+4}\,dx\,dy = -(3)(100)\epsilon_0\left(\frac{1}{2}\right)\ln(x^2+4)\Big|_0^2 = -150\epsilon_0\ln 2 = \underline{-0.92\,\mathrm{nC}}$$

Bài 8:

5.21. Let the surface $y=0$ be a perfect conductor in free space. Two uniform infinite line charges of 30 nC/m each are located at $x=0$, $y=1$, and $x=0$, $y=2$.

a) Let $V=0$ at the plane $y=0$, and find V at $P(1,2,0)$: The line charges will image across the plane, producing image line charges of -30 nC/m each at $x=0$, $y=-1$, and $x=0$, $y=-2$. We find the potential at P by evaluating the work done in moving a unit positive charge from the $y=0$ plane (we choose the origin) to P: For each line charge, this will be:

$$V_P - V_{0,0,0} = -\frac{\rho_l}{2\pi\epsilon_0}\ln\left[\frac{\text{final distance from charge}}{\text{initial distance from charge}}\right]$$

where $V_{0,0,0}=0$. Considering the four charges, we thus have

$$V_P = -\frac{\rho_l}{2\pi\epsilon_0}\left[\ln\left(\frac{1}{2}\right)+\ln\left(\frac{\sqrt{2}}{1}\right)-\ln\left(\frac{\sqrt{10}}{1}\right)-\ln\left(\frac{\sqrt{17}}{2}\right)\right]$$
$$= \frac{\rho_l}{2\pi\epsilon_0}\left[\ln(2)+\ln\left(\frac{1}{\sqrt{2}}\right)+\ln\left(\sqrt{10}\right)+\ln\left(\frac{\sqrt{17}}{2}\right)\right] = \frac{30\times10^{-9}}{2\pi\epsilon_0}\ln\left[\frac{\sqrt{10}\sqrt{17}}{\sqrt{2}}\right]$$
$$= \underline{1.20\,\mathrm{kV}}$$

b) Find $\mathbf{E}$ at P: Use

$$\mathbf{E}_P = \frac{\rho_l}{2\pi\epsilon_0}\left[\frac{(1,2,0)-(0,1,0)}{|(1,1,0)|^2}+\frac{(1,2,0)-(0,2,0)}{|(1,0,0)|^2}\right.$$
$$\left.-\frac{(1,2,0)-(0,-1,0)}{|(1,3,0)|^2}-\frac{(1,2,0)-(0,-2,0)}{|(1,4,0)|^2}\right]$$
$$= \frac{\rho_l}{2\pi\epsilon_0}\left[\frac{(1,1,0)}{2}+\frac{(1,0,0)}{1}-\frac{(1,3,0)}{10}-\frac{(1,4,0)}{17}\right] = \underline{723\,\mathbf{a}_x - 18.9\,\mathbf{a}_y\ \mathrm{V/m}}$$

Bài 9:

5.23. A dipole with $\mathbf{p}=0.1\mathbf{a}_z\ \mu\mathrm{C}\cdot\mathrm{m}$ is located at $A(1,0,0)$ in free space, and the $x=0$ plane is perfectly-conducting.

a) Find V at $P(2,0,1)$. We use the far-field potential for a z-directed dipole:

$$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} = \frac{p}{4\pi\epsilon_0}\frac{z}{[x^2+y^2+z^2]^{1.5}}$$

The dipole at $x=1$ will image in the plane to produce a second dipole of the opposite orientation at $x=-1$. The potential at any point is now:

$$V = \frac{p}{4\pi\epsilon_0}\left[\frac{z}{[(x-1)^2+y^2+z^2]^{1.5}}-\frac{z}{[(x+1)^2+y^2+z^2]^{1.5}}\right]$$

Substituting $P(2,0,1)$, we find

$$V = \frac{.1\times10^{6}}{4\pi\epsilon_0}\left[\frac{1}{2\sqrt{2}}-\frac{1}{10\sqrt{10}}\right] = \underline{289.5\ \mathrm{V}}$$

Bài 10:

6.3. A coaxial conductor has radii $a = 0.8$ mm and $b = 3$ mm and a polystyrene dielectric for which $\epsilon_r = 2.56$. If $\mathbf{P} = (2/\rho)\mathbf{a}_\rho \,\mathrm{nC/m^2}$ in the dielectric, find:

a) $\mathbf{D}$ and $\mathbf{E}$ as functions of ρ: Use

$$\mathbf{E} = \frac{\mathbf{P}}{\epsilon_0(\epsilon_r - 1)} = \frac{(2/\rho)\times 10^{-9}\mathbf{a}_\rho}{(8.85\times 10^{-12})(1.56)} = \underline{\frac{144.9}{\rho}\mathbf{a}_\rho \text{ V/m}}$$

Then

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \frac{2\times 10^{-9}\mathbf{a}_\rho}{\rho}\left[\frac{1}{1.56} + 1\right] = \frac{3.28\times 10^{-9}\mathbf{a}_\rho}{\rho}\ \mathrm{C/m^2} = \underline{\frac{3.28\mathbf{a}_\rho}{\rho}\ \mathrm{nC/m^2}}$$

b) Find V_{ab} and χ_e: Use

$$V_{ab} = \int_3^{0.8} \frac{144.9}{\rho}\,d\rho = 144.9\ln\left(\frac{3}{0.8}\right) = \underline{192\text{ V}}$$

$\chi_e = \epsilon_r - 1 = \underline{1.56}$, as found in part *a*.

Bài 11:

6.5. The surface $x = 0$ separates two perfect dielectrics. For $x > 0$, let $\epsilon_r = \epsilon_{r1} = 3$, while $\epsilon_{r2} = 5$ where $x < 0$. If $\mathbf{E}_1 = 80\mathbf{a}_x - 60\mathbf{a}_y - 30\mathbf{a}_z$ V/m, find:

a) E_{N1}: This will be $\mathbf{E}_1 \cdot \mathbf{a}_x = \underline{80\text{ V/m}}$.

b) $\mathbf{E}_{T1}$. This has components of $\mathbf{E}_1$ *not* normal to the surface, or $\mathbf{E}_{T1} = \underline{-60\mathbf{a}_y - 30\mathbf{a}_z \text{ V/m}}$.

c) $E_{T1} = \sqrt{(60)^2 + (30)^2} = \underline{67.1\text{ V/m}}$.

d) $E_1 = \sqrt{(80)^2 + (60)^2 + (30)^2} = \underline{104.4\text{ V/m}}$.

e) The angle θ_1 between $\mathbf{E}_1$ and a normal to the surface: Use

$$\cos\theta_1 = \frac{\mathbf{E}_1\cdot\mathbf{a}_x}{E_1} = \frac{80}{104.4} \ \Rightarrow\ \theta_1 = \underline{40.0^\circ}$$

f) $D_{N2} = D_{N1} = \epsilon_{r1}\epsilon_0 E_{N1} = 3(8.85\times 10^{\;\,12})(80) = \underline{2.12\,\mathrm{nC/m^2}}$.

g) $D_{T2} = \epsilon_{r2}\epsilon_0 E_{T1} = 5(8.85\times 10^{-12})(67.1) = \underline{2.97\,\mathrm{nC/m^2}}$.

h) $\mathbf{D}_2 = \epsilon_{r1}\epsilon_0 E_{N1}\mathbf{a}_x + \epsilon_{r2}\epsilon_0\mathbf{E}_{T1} = \underline{2.12\mathbf{a}_x - 2.66\mathbf{a}_y - 1.33\mathbf{a}_z\ \mathrm{nC/m^2}}$.

i) $\mathbf{P}_2 = \mathbf{D}_2 - \epsilon_0\mathbf{E}_2 = \mathbf{D}_2\left[1 - (1/\epsilon_{r2})\right] = (4/5)\mathbf{D}_2 = \underline{1.70\mathbf{a}_x - 2.13\mathbf{a}_y - 1.06\mathbf{a}_z\ \mathrm{nC/m^2}}$.

j) the angle θ_2 between $\mathbf{E}_2$ and a normal to the surface: Use

$$\cos\theta_2 = \frac{\mathbf{E}_2\cdot\mathbf{a}_x}{E_2} = \frac{\mathbf{D}_2\cdot\mathbf{a}_x}{D_2} = \frac{2.12}{\sqrt{(2.12)^2 = (2.66)^2 + (1.33)^2}} = .581$$

Thus $\theta_2 = \cos^{-1}(.581) = \underline{54.5^\circ}$.

Bài 12:

6.7. Two perfect dielectrics have relative permittivities $\epsilon_{r1} = 2$ and $\epsilon_{r2} = 8$. The planar interface between them is the surface $x - y + 2z = 5$. The origin lies in region 1. If $\mathbf{E}_1 = 100\mathbf{a}_x + 200\mathbf{a}_y - 50\mathbf{a}_z$ V/m, find $\mathbf{E}_2$: We need to find the components of $\mathbf{E}_1$ that are normal and tangent to the boundary, and then apply the appropriate boundary conditions. The normal component will be $E_{N1} = \mathbf{E}_1 \cdot \mathbf{n}$. Taking $f = x - y + 2z$, the unit vector that is normal to the surface is

$$\mathbf{n} = \frac{\nabla f}{|\nabla f|} = \frac{1}{\sqrt{6}} [\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z]$$

This normal will point in the direction of increasing f, which will be away from the origin, or into region 2 (you can visualize a portion of the surface as a triangle whose vertices are on the three coordinate axes at $x = 5$, $y = -5$, and $z = 2.5$). So $E_{N1} = (1/\sqrt{6})[100 - 200 - 100] = -81.7\,\mathrm{V/m}$. Since the magnitude is negative, the normal component points into region 1 from the surface. Then

$$\mathbf{E}_{N1} = -81.65 \left(\frac{1}{\sqrt{6}}\right) [\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z] = -33.33\mathbf{a}_x + 33.33\mathbf{a}_y - 66.67\mathbf{a}_z \text{ V/m}$$

Now, the tangential component will be $\mathbf{E}_{T1} = \mathbf{E}_1 - \mathbf{E}_{N1} = 133.3\mathbf{a}_x + 166.7\mathbf{a}_y + 16.67\mathbf{a}_z$. Our boundary conditions state that $\mathbf{E}_{T2} = \mathbf{E}_{T1}$ and $\mathbf{E}_{N2} = (\epsilon_{r1}/\epsilon_{r2})\mathbf{E}_{N1} = (1/4)\mathbf{E}_{N1}$. Thus

$$\mathbf{E}_2 = \mathbf{E}_{T2} + \mathbf{E}_{N2} = \mathbf{E}_{T1} + \frac{1}{4}\mathbf{E}_{N1} = 133.3\mathbf{a}_x + 166.7\mathbf{a}_y + 16.67\mathbf{a}_z - 8.3\mathbf{a}_x + 8.3\mathbf{a}_y - 16.67\mathbf{a}_z$$
$$= \underline{125\mathbf{a}_x + 175\mathbf{a}_y \text{ V/m}}$$

Bài 13:

6.8. Region 1 ($x \geq 0$) is a dielectric with $\epsilon_{r1} = 2$, while region 2 ($x < 0$) has $\epsilon_{r2} = 5$. Let $\mathbf{E}_1 = 20\mathbf{a}_x - 10\mathbf{a}_y + 50\mathbf{a}_z$ V/m.

a) Find $\mathbf{D}_2$: One approach is to first find $\mathbf{E}_2$. This will have the same y and z (tangential) components as $\mathbf{E}_1$, but the normal component, E_x, will differ by the ratio $\epsilon_{r1}/\epsilon_{r2}$; this arises from $D_{x1} = D_{x2}$ (normal component of $\mathbf{D}$ is continuous across a non-charged interface). Therefore $\mathbf{E}_2 = 20(\epsilon_{r1}/\epsilon_{r2})\,\mathbf{a}_x - 10\,\mathbf{a}_y + 50\,\mathbf{a}_z = 8\,\mathbf{a}_x - 10\,\mathbf{a}_y + 50\,\mathbf{a}_z$. The flux density is then

$$\mathbf{D}_2 = \epsilon_{r2}\epsilon_0\mathbf{E}_2 = 40\epsilon_0\,\mathbf{a}_x - 50\epsilon_0\,\mathbf{a}_y + 250\epsilon_0\,\mathbf{a}_z = \underline{0.35\,\mathbf{a}_x - 0.44\,\mathbf{a}_y + 2.21\,\mathbf{a}_z \text{ nC/m}^2}$$

b) Find the energy density in both regions: These will be

$$w_{e1} = \frac{1}{2}\epsilon_{r1}\epsilon_0\mathbf{E}_1 \cdot \mathbf{E}_1 = \frac{1}{2}(2)\epsilon_0 \left[(20)^2 + (10)^2 + (50)^2\right] = 3000\epsilon_0 = \underline{26.6 \text{ nJ/m}^3}$$

$$w_{e2} = \frac{1}{2}\epsilon_{r2}\epsilon_0\mathbf{E}_2 \cdot \mathbf{E}_2 = \frac{1}{2}(5)\epsilon_0 \left[(8)^2 + (10)^2 + (50)^2\right] = 6660\epsilon_0 = \underline{59.0 \text{ nJ/m}^3}$$

Bài 14:

6.9. Let the cylindrical surfaces $\rho = 4$ cm and $\rho = 9$ cm enclose two wedges of perfect dielectrics, $\epsilon_{r1} = 2$ for $0 < \phi < \pi/2$, and $\epsilon_{r2} = 5$ for $\pi/2 < \phi < 2\pi$. If $\mathbf{E}_1 = (2000/\rho)\mathbf{a}_\rho$ V/m, find:

a) $\mathbf{E}_2$: The interfaces between the two media will lie on planes of constant ϕ, to which $\mathbf{E}_1$ is parallel. Thus the field is the same on either side of the boundaries, and so $\underline{\mathbf{E}_2 = \mathbf{E}_1}$.

b) the total electrostatic energy stored in a 1m length of each region: In general we have $w_E = (1/2)\epsilon_r\epsilon_0 E^2$. So in region 1:

$$W_{E1} = \int_0^1 \int_0^{\pi/2} \int_4^9 \frac{1}{2}(2)\epsilon_0 \frac{(2000)^2}{\rho^2}\, \rho\, d\rho\, d\phi\, dz = \frac{\pi}{2}\epsilon_0 (2000)^2 \ln\left(\frac{9}{4}\right) = \underline{45.1\ \mu\text{J}}$$

In region 2, we have

$$W_{E2} = \int_0^1 \int_{\pi/2}^{2\pi} \int_4^9 \frac{1}{2}(5)\epsilon_0 \frac{(2000)^2}{\rho^2}\, \rho\, d\rho\, d\phi\, dz = \frac{15\pi}{4}\epsilon_0 (2000)^2 \ln\left(\frac{9}{4}\right) = \underline{338\ \mu\text{J}}$$

Bài 15:

6.10. Let $S = 100\,\text{mm}^2$, $d = 3$ mm, and $\epsilon_r = 12$ for a parallel-plate capacitor.

a) Calculate the capacitance:

$$C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{12\epsilon_0 (100 \times 10^{-6})}{3 \times 10^{-3}} = 0.4\epsilon_0 = \underline{3.54\ \text{pf}}$$

b) After connecting a 6 V battery across the capacitor, calculate E, D, Q, and the total stored electrostatic energy: First,

$$E = V_0/d = 6/(3 \times 10^{-3}) = \underline{2000\ \text{V/m}}, \quad \text{then} \quad D = \epsilon_r\epsilon_0 E = 2.4 \times 10^4 \epsilon_0 = \underline{0.21\ \mu\text{C/m}^2}$$

The charge in this case is

$$Q = \mathbf{D} \cdot \mathbf{n}|_s = DA = 0.21 \times (100 \times 10^{-6}) = 0.21 \times 10^{-4}\ \mu\text{C} = \underline{21\ \text{pC}}$$

Finally, $W_e = (1/2)QV_0 = 0.5(21)(6) = \underline{63\ \text{pJ}}$.

Bài 16:

Let $\epsilon_{r1} = 2.5$ for $0 < y < 1$ mm, $\epsilon_{r2} = 4$ for $1 < y < 3$ mm, and ϵ_{r3} for $3 < y < 5$ mm. Conducting surfaces are present at $y = 0$ and $y = 5$ mm. Calculate the capacitance per square meter of surface area if: a) ϵ_{r3} is that of air; b) $\epsilon_{r3} = \epsilon_{r1}$; c) $\epsilon_{r3} = \epsilon_{r2}$; d) region 3 is silver: The combination will be three capacitors in series, for which

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\epsilon_{r1}\epsilon_0(1)} + \frac{d_2}{\epsilon_{r2}\epsilon_0(1)} + \frac{d_3}{\epsilon_{r3}\epsilon_0(1)} = \frac{10^{-3}}{\epsilon_0}\left[\frac{1}{2.5} + \frac{2}{4} + \frac{2}{\epsilon_{r3}}\right]$$

So that

$$C = \frac{(5 \times 10^{-3})\epsilon_0\epsilon_{r3}}{10 + 4.5\epsilon_{r3}}$$

Evaluating this for the four cases, we find a) $C = \underline{3.05\,\text{nF}}$ for $\epsilon_{r3} = 1$, b) $C = \underline{5.21\,\text{nF}}$ for $\epsilon_{r3} = 2.5$, c) $C = \underline{6.32\,\text{nF}}$ for $\epsilon_{r3} = 4$, and d) $C = \underline{9.83\,\text{nF}}$ if silver (taken as a perfect conductor) forms region 3; this has the effect of removing the term involving ϵ_{r3} from the original formula (first equation line), or equivalently, allowing ϵ_{r3} to approach infinity.

Bài 17:

6.17. Two coaxial conducting cylinders of radius 2 cm and 4 cm have a length of 1m. The region between the cylinders contains a layer of dielectric from $\rho = c$ to $\rho = d$ with $\epsilon_r = 4$. Find the capacitance if

a) $c = 2$ cm, $d = 3$ cm: This is two capacitors in series, and so

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2\pi\epsilon_0}\left[\frac{1}{4}\ln\left(\frac{3}{2}\right) + \ln\left(\frac{4}{3}\right)\right] \Rightarrow C = \underline{143\,\text{pF}}$$

b) $d = 4$ cm, and the volume of the dielectric is the same as in part a: Having equal volumes requires that $3^2 - 2^2 = 4^2 - c^2$, from which $c = 3.32$ cm. Now

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2\pi\epsilon_0}\left[\ln\left(\frac{3.32}{2}\right) + \frac{1}{4}\ln\left(\frac{4}{3.32}\right)\right] \Rightarrow C = \underline{101\,\text{pF}}$$

Bài 18:

6.19. Two conducting spherical shells have radii $a = 3$ cm and $b = 6$ cm. The interior is a perfect dielectric for which $\epsilon_r = 8$.

a) Find C: For a spherical capacitor, we know that:

$$C = \frac{4\pi\epsilon_r\epsilon_0}{\frac{1}{a} - \frac{1}{b}} = \frac{4\pi(8)\epsilon_0}{\left(\frac{1}{3} - \frac{1}{6}\right)(100)} = 1.92\pi\epsilon_0 = \underline{53.3\,\text{pF}}$$

b) A portion of the dielectric is now removed so that $\epsilon_r = 1.0$, $0 < \phi < \pi/2$, and $\epsilon_r = 8$, $\pi/2 < \phi < 2\pi$. Again, find C: We recognize here that removing that portion leaves us with two capacitors in parallel (whose C's will add). We use the fact that with the dielectric *completely* removed, the capacitance would be $C(\epsilon_r = 1) = 53.3/8 = 6.67\,\text{pF}$. With one-fourth the dielectric removed, the total capacitance will be

$$C = \frac{1}{4}(6.67) + \frac{3}{4}(53.4) = \underline{41.7\,\text{pF}}$$